

Identities in plactic monoids and monoids of upper-triangular tropical matrices

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Abstract

We use the combinatorics of Young tableaux to prove the plactic monoid of infinite rank does not satisfy a non-trivial identity, by showing that the plactic monoid of rank n cannot satisfy a non-trivial identity of length less than or equal to n . A new identity is then proven to hold for the monoid of $n \times n$ upper-triangular tropical matrices. Finally, a straightforward embedding is exhibited of the plactic monoid of rank 3 into the direct product of two copies of the monoid of 3×3 upper-triangular tropical matrices, giving a new proof that the plactic monoid of rank 3 satisfies a non-trivial identity.

References

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